DBM-003-1163004 Seat No. ______

M. Sc. (Sem. III) (CBCS) Examination

June - 2022

Mathematics : Course No. 3004

(Discrete Mathematics)

Faculty Code: 003

Subject Code: 1163004

Time : **2.30** Hours] [Total Marks : **70**

Instructions:

(1) There are ten questions.

(2) Answer any five of them.

(3) Each question carries 14 marks.

1 Answer the following : **7×2=14**

(a) Define: Minterm and Complemented lattice.

(b) Define with example: Isomorphism of Monoids.

(c) Draw: Hasse diagram for (D_{30}, R).

(d) Let $M_1 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ $M_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$. Find $M_1 \odot M_2$

(e) Define: Lattice, with example.

(f) Define: (1) Sub-Lattice (2) Distributive lattice.

(g) Define a Congruence relation on a Semigroup.

(h) Define: Poset with example.

2 Answer the following : **7×2=14**

(1) Define: Machine congruence on a finite state machine.

(2) Define: Phrase structure grammar.

(3) State: Kleen's Theorem.

(4) Define: Language of a Moore machine.

(5) Define the term: Proposition with example.

(6) Make a truth table for the statement: $(p \wedge q) \vee (\sim p)$.

(7) Make truth tables for the statements: (i) $p \wedge q$ (ii) $p \vee q$.

3 Answer the following questions: **2×7=14**

(a) Let R be a relation defined on A and $|A| = n$. Prove that,

$R^{\infty} = R \cup R^2 \cup R^3 \cup .... \cup R^n$.

(b) Define a Modular lattice. Let $(L, \leq)$ be lattice. Then $(L, \leq)$ is Modular lattice if and only if the following holds: "If M is any Sublattice of $(L, \leq)$. Then M is not isomorphic to the Pentagon lattice."

4 Answer the following questions: **2×7=14**

(a) Let G be a group and S be a normal subgroup of G. Let R be a relation on G by aRb if and only if $ab^{-1} \in S$. Prove that, R is a congruence relation on G.

(b) Let $p(x,y,z) = (x \wedge y) \vee (y \wedge z')$. Determine the function $f : B_3 \to B$ induced by $p(x,y,z)$.

5 Answer the following questions: **2×7=14**

(a) Let p, q be any proposition or statement. Prove that, each of the following compound statements are tautology:

(i) $p \wedge q \Rightarrow p$

(ii) $p \Rightarrow p \vee q$

(iii) $\sim p \Rightarrow (p \Rightarrow q)$

(iv) $\sim (p \Rightarrow q) \Rightarrow p$

(v) $[p \wedge (p \Rightarrow q)] \Rightarrow q$

(vi) $[p \wedge (p \Rightarrow q)] \Rightarrow p$

(b) For the languages given in (i) and (ii) below, construct a phrase structure grammar G such that $L(G) = L$.

(i) $L = \{a^n b^m / n \geq 1, m \geq 3\}$ and

(ii) $L = \{x^n y^m / n \geq 2, m \geq 0 \text{ and even}\}$

6 Answer the following questions: **2×7=14**

(a) Define: Lexicographic order. Let $n \geq 1$. Let (L, $\leq$) be a finite Boolean algebra. Prove that, the number of atoms of (L, $\leq$) is same as number of co-atoms of (L, $\leq$).

(b) State and prove: Pumping lemma.

7 Answer the following questions: **2×7=14**

(a) State and prove: Fundamental theorem of Homomorphism of semigroups.

(b) Describe steps of Warshall's Algorithm for finding W_k from $W_{k-1}, k \in \{1, 2, ..., n\}$. Also using them find R^{∞} for A = {1, 2, 3, 4} with R = {(1, 2), (2, 3), (3, 2), (3, 4)}.

8 Answer the following questions: **2×7=14**

(a) Define atom. Let (L, $\leq$) be a finite Boolean algebra. Let $a \in L, a \neq 0$. Let $\{a_1, a_2, a_3, ... , a_m\}$ be the set of all atoms of (L, $\leq$) such that $a_i \leq a$ for each $i \in \{1, 2, ..., m\}$. Prove that, $a = a_1 \vee a_2 \vee a_3 \vee ... \vee a_m$

(b) Define: GLB and LUB of a subset of (P, $\leq$). Let $(L_i \leq_i)$ be lattices for each $i \in \{1, 2, ... , n\}$. Let $L = L_1 \times L_2 \times .... \times L_n$ be the Cartesian product of $L_1, L_2, ..., L_n$. Let $\leq$ be the product partial order on L. Prove that, $(L = L_1 \times L_2 \times .... \times L_n, \leq)$ is also a lattice.

9 Answer the following questions: **2×7=14**

(a) Let R is an equivalence relation on A = {1, 2, 3, 4, 5} determined by the partition P_1 of A whose members are {1, 2}, {3, 4}, {5} and S is another equivalence relation on A determined by the partition P_2 of A whose members are {1}, {2}, {3}, {4, 5}. Find $(R \cup S)^{\infty}$ using:

(i) Graphical Method (ii) Matrix Method

(b) Let $f_1 : B_2 \to B$ be a Boolean function with $S(f_1) = \{00, 01, 10\}$ and let $f_2 : B_3 \to B$ be a Boolean function with $S(f_2) = \{000, 001, 011, 010\}$. Construct Karnaugh maps for both f_1 and f_2. Also find the Boolean expressions for both of them.

10 Answer the following questions: **2×7=14**

(a) Prove that $(\mathbb{N}, R)$ is distributive lattice, where R is divisibility relation on $\mathbb{N}$.

(b) Let *p, q* be propositions. Prove that the following statements hold:

(i) $(p \Rightarrow q) \equiv (\sim p) \vee q$

(ii) $(p \Rightarrow q) \equiv \sim q \Rightarrow \sim p$

(iii) $\sim (p \Rightarrow q) \equiv (p \wedge \sim q)$

(iv) $\sim (p \Leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

(v) $(p \Leftrightarrow q) \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$

(vi) $\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$

(vii) $\sim (\sim p) \equiv p$

(viii) $\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$

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